plan followed in the problems that the student has solved in Arithmetic, Algebra, and Geometry. It should not be forgotten, however, that when we apply the principles of Trigonometry to the solution of practical problems,—engineering problems, for instance,—it is usually necessary to use data which have feen found by actual measurement and therefore are subject to error. For instance, if the length of a line is measured by a steel tape, account must be taken of the expansion due to heat as well as the sagging of the tape under various tensions. And in making several measurements one should carefully see that they are made with about the same precision. Thus, it would be folly to measure one side of a triangle with much greater care than another; for, in combining these measurements in a calculation, the result would at best be no more accurate than the worst measurement. Similarly, the angles of a triangle should be measured with the same care as the sides. The number of significant figures in a measurement are supposed to indicate the care that was intended when the measurement was made. In ordinary engineering practice only the first three or four significant figures of the measurement are not subject to error. It is therefore evident that the use of five or six place tables in calculations involving these measurements introduces an unnecessary refinement and merely adds to the labor without making the results more accurate than they would be if four place tables had been used. In a large number of cases three place tables are accurate enough.

At regular intervals the student should be required to write down from memory a list of all the fundamental formulas studied.

Brief notes on the history of the science should be given by the teacher as opportunity offers.

The computation of logarithms and of the trigonometric functions from series, De Moivre's Theorem, and the hyperbolic functions, do not properly belong to a first course in Trigonometry.

FACSIMILE EDITIONS OF JOHN BOLYAI'S SCIENCE ABSOLUTE OF SPACE.

By GEORGE BRUCE HALSTED.

At last, any one has a chance to see just how looked the most extraordinary two dozen pages in the history of human thought.

How John Bolyai himself looked, the world can never know, for before it woke up to his genius he was dead of disgust and so covered with oblivion that no picture of him remains, though his love child Dyonis, son of Rosa Orbán, still lives. His eyes were blue. So much I read in his passport

at Maros-Vásárhely, where I obtained from the college in which John's father, Farkas, was professor, its copy of the priceless gem now reproduced in facsimile by Stephen Biás de Ders (Dersi Biás István).

This facsimile is the more welcome because in my translation, now in its fourth edition, are reproduced entire in Japan in English, I put Bolyai's symbols into my own words in a terminology I had already used in translating Lobachevsky, and sacrificed to uniformity the marked distinction in notation between these two independent creators of the new universe. Both recognize that through a point A outside of a straight BC there may be an infinity of straights coplanar with BC but nowhere meeting it. This was also discovered independently by Philip Kelland, Cambridge senior wrangler and tutor of the great Sylvester, whose real name was not Sylvester but plain James Joseph. Now John Bolyai and Kelland, adhering strictly to Euclid's definition of parallels, called all these intersecting straights parallel to BC. Kelland made no further distinction among them, and his work remained sterile. The others both recognized the all-importance of a boundary line, and to these boundary lines Lobachevsky restricted the application of the word parallel; so that through every point he has two intersecting parallels to the same straight, one parallel to it toward one end of it, the other parellel to it toward the other end of it, making of parallelism a *sensed* relation. For the remaining infinity of the Bolyai and Kelland parallels Lobachevsky has no word. For them I have adopted the term *ultra-parallel.* In this terminology each parallel to a straight meets it at infinity. Two straights parallel to each other have a figurative point in common. But two straights ultra-parallel to each other, though coplanar, have not even a figurative point in common. They do not meet even at infinity. Two straights parallel in opposite senses to the same straight intersect. Two straights parallel to the same straight in the same sense are parallel to one another.

Into this terminology I translated or rather paraphrased John Bolyai, and the world has accepted it. He largely wrote in symbols of his own make, which never have been used by any one else.

His symbol for that half of the straight AB which commences at the point A and contains the point B, I translate *the ray AB.*

His first section begins: "If the ray AM is not cut by the coplanar ray BN, but is cut by every ray BP within the angle ABN, this is designated by $BN \,|||\, AM$." His symbol $|||$ I replaced by $\|$ and translate *parallel.* But for Bolyai himself it meant *asymptote.* And no one word could have expressed more of the new thought. That two rays can be asymptotes to one another seemed so strange an idea we cannot sufficiently marvel at the courage, the nerve, the fineness of our young Magyar.

B. S. Teubner, of Leipzig, has on sale 300 copies of the *Ders Facsimile.* But let those who on it see the date of the original, 1832, not forget the letter I saw at Maros-Vásárhely and was the first to publish, making the date

1823 ever memorable. On its publication thus in America Charles S. Peirce wrote in *The Nation,* March 17, 1892, p. 212, in a review of Halsted's Bolyai:

> There is a winningly enthusiastic letter from Bolyai János to his father, telling him of the great step. He says: "I have discovered such magnificent things that I am myself astonished at them. It would be damage eternal if they were lost. When you see them, my father, you will yourself acknowledge it. At present I can not say more than that from nothing I have created a wholly new world."

Ten years later this letter was published in Hungary in Magyar and Latin. Later came the establishment of the great Bolyai prize (Prix Bolyai) by the Hungarian Academy of Sciences, a prize of ten thousand crowns founded on the occasion of the hundredth anniversary of the birth of John Bolyai "to perpetuate the memory of this illustrious scientist," whose very grave was lost, but for one woman, who told where to erect the belated granite shaft now dedicated to dauntless genius. Before the prize was first given I ventured in *Science,* as a bit of prophecy, to predict it would go to Poincaré. It did. Now it is again to be adjudged in 1910.

Attempts are being made to fix the site of Bolyai's house, No. 1004, burned in 1876. Meanwhile the quaint house of his father Farkas, redolent with memories of Hungary's wonder-child, in which I was entertained so charmingly by Professor Koncz, who died in 1906, has been torn down to make way for a new street. Franz Schmidt of Budapesth has also passed away, who on the banks of the Danube confided to me the tales his own father told him of the fiery young Captain Bolyai, whom he saw, with his treasured Damascus sword, hack off an iron spike driven into his door-post, and who, another Ivanhoe, challenged by 13 Austrian cavalry officers at once, discomfited in succession all the challengers, a swordsman so redoubtable he could have held at bay those kindred spirits D'Artagnan, Cyrano and Captain Fracasse. Cyrano composed poetry while swording his man. Bolyai played his marvellous violin between bouts while sabreing his 13 cavalry officers. But what if he had taken it into that wonderful head of his to challenge the whole Austrian army in squads of thirteen?

His violin he expressly mentions in his will. I know not the fate of Damascus blade; but now we are given in facsimile the still more trenchant instrument wherewith he severed the bonds in which throughout the ages Euclid had held captive the mind of man.